Robust models for simultaneous open pit and underground mines

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Abstract: Mining planning is a central problem in the mining business with the goal of setting high revenued exploitation agendas. However, current models lack robustness: they do not consider uncertainty of the future, so the plans are, indeed, recalculated every year according to the new information.

This work presents some approaches to solve the above-mentioned problem: at first the current models are exposed, stochastic programming is used then to set up new models considering uncertainity. As a second contribution, the new models also generalize the current ones because they allow combined exploitation, setting up plans where underground and open pit technologies can be used simultaneously.

Moreover, after a deeper revision of the current models and the stochastic programs mentioned, a second approach is exposed. This model, in the hope of more robust mining schedules, searches for schedules satisfying demands and capacities maximizing the residual ore for future, instead of the NPV value used in the previous ones. Finally, a heuristic is presented to solve this model.

Key-words: mining scheduling, robustness, stochastic programming.

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Modèles robustes pour mines souterraines et à ciel ouvert

Résumé : La planification des exploitations minières est une problematique essentielle dans les entreprises du secteur, qui vise à trouver des programmes d'exploitation aux profits plus élevés. Cependant, les modèles actuels ne sont pas fiables: Ils ne prennent pas en compte les incertitudes de l'avenir. Les planifications doivent donc être recalculées annuellement afin de considérer les nouvelles informations.

Ce travail présente de nouvelles perspectives pour resoudre le problème. En un premier temps, on présente les modèles actuels pour ensuite, à travers la programation stochastique, proposer de nouveaux modèles qui considèrent le facteur de l'incertitude et qui géneralisent les anterieurs puisqu'ils sont capables de traiter les exploitations combinées. En effet, ils mettent en place des plans où les méthodes des exploitations souterraines et à ciel ouvert peuvent être utilisées simultanement.

Finalment, après une revision plus profonde des modèles considérés, une deuxième approche est présentée. Ce modèle, visant une majeure robustesse, cherche des planifications minières qui satisfassent les demandes et capacités conservant pour le futur la plus grande quantité de minéral non-exploité au lieu de maximiser la VPN, comme faisaient les modèles précedants. Une heuristique est présentée alors pour resoudre ce modèle.

Mots-clés : Planification des mines, robustesse, programmation stochastique.

Contents

1 Introduction

The copper mining industry has been important in Chile even before the country existed. The mine *Chuquicamata* (Chile) was exploited by the Incas when no European feet has touched the continent yet, and even nowadays it exists and is the biggest open pit mine in the world. However, and despite the importance of the industry (which represents about 40% of the Gross National Product), mathematical problems concerning the mining processes are very recent, but, on the other hand, quite diverse. This work is focused on *mining scheduling* which goal is to fix long term exploitation agendas for a mine.

There are two kinds of copper mines: open pit mines (where exploitation is performed on the surface), and the others, the underground mines. Open pit mining models have been much more (historically) developed than underground mining and the *battle horse* to set up high profiting open pit exploitation plans has been the *ultimate pit problem*, which corresponds to determine the pit with higher economic content under the assumption of infinite capacity. Of course in the real world this hypothesis does not hold, but the existence of an efficient algorithm, proposed by Lerchs and Grossman in the 60's [4], to solve it has made this problem the kernel of almost every heuristic trying to approach the true problem in real world instances.

Underground mining optimization models are so recent that there is not commercial software implementing them yet, and only (several) heuristics has been used to set mining schedules in real mines [5]. Nevertheless, these models have the advantage that they solve a more realistic problem, including capacity constraints. On the other hand, they need to do some preprocessing to make the information more aggregated and are linked with one specific underground exploitation mining technology.

The mining schedules obtained by the current models are quite non-robust. Once the conditions (on price, demand, as well as the expected ore grades) change, they become suboptimal and therefore, must be recalculated. A major goal of this work is to introduce robustness in the planning process. This is done in two different ways: using stochastic programming and setting up models oriented not to NPV maximization, but to capacity/demand satisfaction. As an additional contribution, this models are developed in such a way they allow both underground and open pit technologies to coexist.

This report is organized as follows. At first, current (separated) open pit and underground models are revised. After that, stochastic elements are exposed in order to introduce some elementary concepts of the area and how robustness appears in stochastic programming. Next, the exposed elements are applied to the case of mining scheduling: the major sources of uncertainty in the problem and two stochastic programming models are developed.

The above mentioned models are stochastic extensions of NPV models, the most widely approach to the mine scheduling problem. However, there are others ways in the quest of *robust* models for mine scheduling. That is the reason for setting up a different kind of model oriented to capacity and demand satisfaction instead of NPV maximization. Finally, a heuristic to solve this problem is proposed.

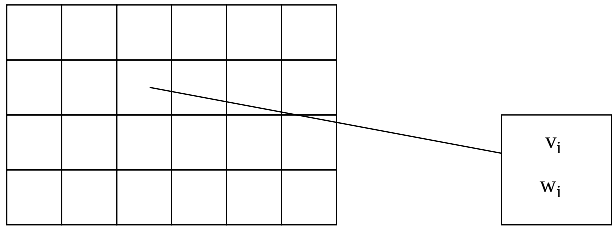

Figure 1: Block model

2 Current models

For the sake of simplicity, all the concepts here are presented in a 2D-world.

2.1 Open pit mines

Current approaches to the open pit mine problem are based on the block model: a imaginary partition of the mine into rectangles. For the i-th block, two quantities are considered (see Fig 1) :

1. its *economic value* $v_i \in \mathbb{R}$, which can be negative if the value of its contained ore is not enough to pay the extraction of the block,
2. $w_i \in \mathbb{R}^+$, the *weight* of the block.

Associated with the block model there is also a graph $G = (E, V)$ (Fig. 2), such that E is the set of blocks and

$$(i, j) \in V \;\Leftrightarrow\; j \text{ must be exploited before } i.$$

Figures 3 and 4 show feasible and infeasible pits (exploitation states), respectively. Three other concepts are shown in these pictures also:

- The initial surface does not need to be flat.
- Because the world does not end where the block model does, the top corner blocks (grey) cannot be removed (surface smoothness ought to be hold).
- For open pit planning purposes and due to precedence constraints, there are many blocks that will never be extracted.

In Figure 4 the exploitation of circle-marked blocks violates precedence of the ones with the crossed pattern.

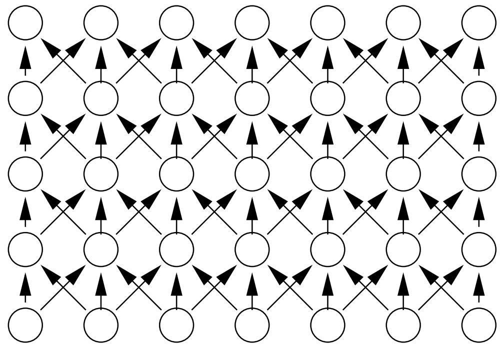

Figure 2: Precedence graph

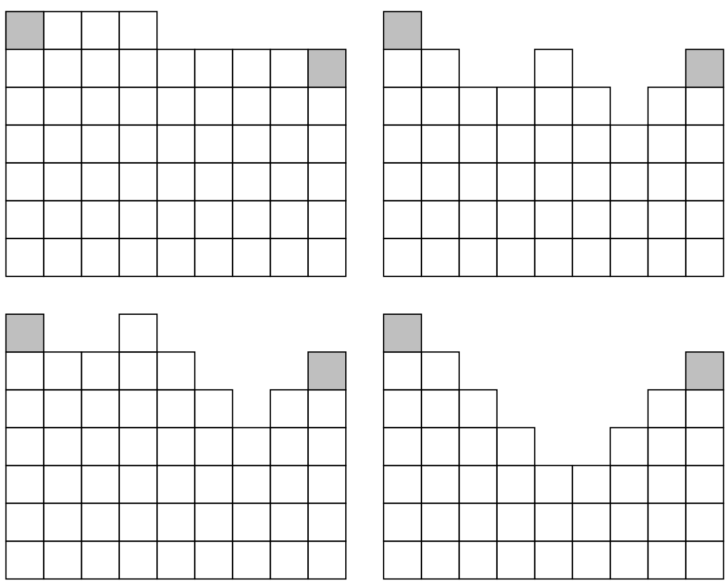

Figure 3: Feasible pits

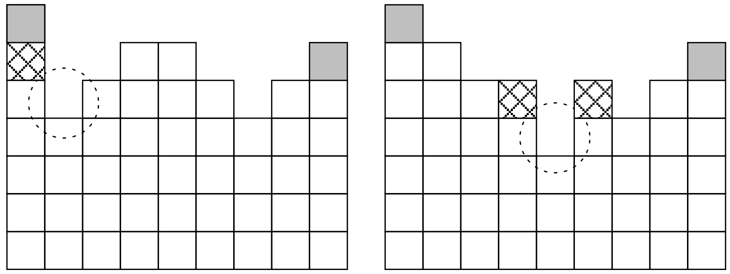

Figure 4: Infeasible pits

2.1.1 The ultimate pit problem

It corresponds to find the subset S of E maximizing the sum of the values of the blocks contained in, and holding precedence restrictions of G. More precisely, if $x_i = \mathbb{I}_S(i)$ (the index function of S), then $\sum_{i \in S} v_i = \sum_{i \in E} v_i x_i$, and the problem can be written as

$$
\begin{array}{lll}
(UP) & \max\limits_{x} & \sum\limits_{i \in E} v_i x_i \\
& \text{s.t.} & x_i \leq x_j \;\; (\forall (i,j) \in E) \\
& & x \in \{0,1\}^{|V|}
\end{array}
$$

This problem is called the *ultimate-pit problem* [3], because its solution is the maximum pit (in inclusion sense) to be exploited, when capacity constraints are not present; but it is also known as the *maximum closure graph* problem, which can be reduced to MAXFLOW-MINCUT [6]. Despite of that, commercial software implements specific algorithms to solve it. The most widely used is the Lerchs & Grossman algorithm [4].

Notice that the w_i coefficients do not appear in this model, because this model does not include capacity. However, in the real world there exist capacity constraints (of extraction and processing) that avoid the whole solution of (UP) become extracted at once. This means that one has to decide which portion of the solution S has to be exploited first, and thus *time* appears in the problem: if block i is sold in the next period, the profit (measured in today's money) obtained by the company will be only αv_i, where $\alpha \in (0,1)$ corresponds to the *opportunity cost.*[1] Of course, different *sequences of exploitation* yield different net present values (NPV), so the problem becomes even harder: it is not enough to decide *what* to exploit, but also *when* to do it.

[1] See the appendix for a better understanding of this concept.

2.1.2 Open pit sequencing model

Because of the previous elements, it is necessary to consider:

- A discount rate of $\alpha \in (0,1)$.
- $T \in I\!N$ periods of time.
- Decision variables:

$$x_{it} = \begin{cases} 1 & \text{Block } i \text{ is extracted at period } t \\ 0 & \text{Otherwise.} \end{cases}$$

- C_t : the capacity at period t.

With these definitions, the *open pit sequencing problem* could be set as

$$\begin{aligned} \text{(OPS)} \ \max_{\vec{x}} \quad \text{NPV} = \ & \sum_{i,t} \alpha^t \, v_i \, x_{it} \\ & \sum_{s \le t} (x_{i,s} - x_{j,s}) \ \le 1 \ (\forall (i,j) \in E)(\forall t) \\ & \sum_i w_i \, x_{it} \ \le \ C_t \ (\forall t) \\ & \sum_t x_{i,t} \ \le \ 1 \ (\forall i) \\ & x_{i,t} \in \{0,1\} \end{aligned}$$

Because of the huge number of variables, classical approaches to solve OPS could easily become an infeasible way: $|V|$ is often over 1 million, so, just for a 10 periods of time instance, (OPS) becomes a 10.000.000 binary linear optimization problem. Due to this fact, many heuristics have been developed in order to find good feasible sequences of (OPS). A couple of examples are available in [8] and [9].

About the theoretical complexity, it is possible to prove that the easier and simplified version (which appears in the limits cases of $\alpha = 0, 1$):

$$\begin{aligned} (UPC) \ \max_x \quad & \sum_{i \in E} v_i x_i \\ \text{s.t.} \quad & x_i \le x_j \ (\forall (i,j) \in E) \\ & \sum_i w_i x_i \le C \\ & x \in \{0,1\}^{|V|} \end{aligned}$$

is NP-Hard [8].

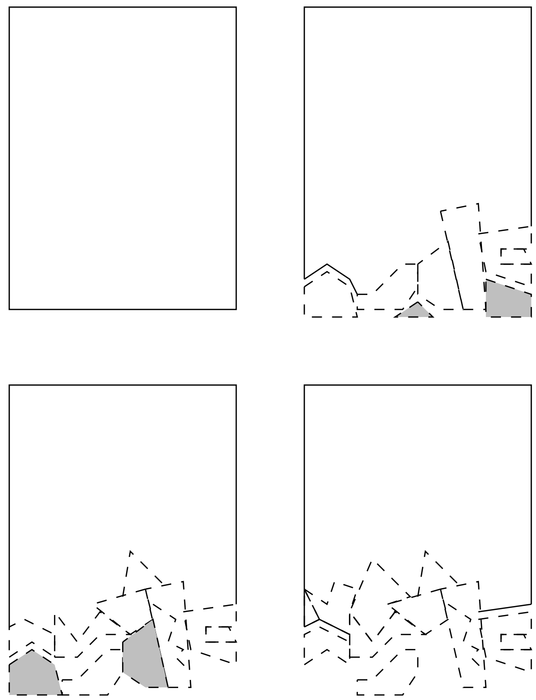

Figure 5: Individual block exploitation.

2.2 Underground mines

Programming models for underground mining (UM from now) are very recent and adapted to one specific kind of exploitation technology, named *Block Caving* [5]. However, this means that to set an instance of these models some important preprocessing is required.

In block caving terminology, blocks are called *cells* and are grouped into big blocks, which are exploited "independently" from each others. The exploitation of a specific block starts at the block's base or *floor*, where fractures are performed, so the material collapses, flows down due to gravity force and is collected. This collection produces additional empty space, so bigger efforts appear in the rock which collapses again and so on. Figure 5 shows it very schematically (grey portions are removed at each stage).

Blocks are grouped into several sectors, which can be placed at different levels, so, due to the breakage of material, any interaction between levels must be avoided. More precisely, if one block is sunk at time t, no others localized *over it* can be exploited after this period. Block j is over block i if its base intersects the cone starting at block i's base.

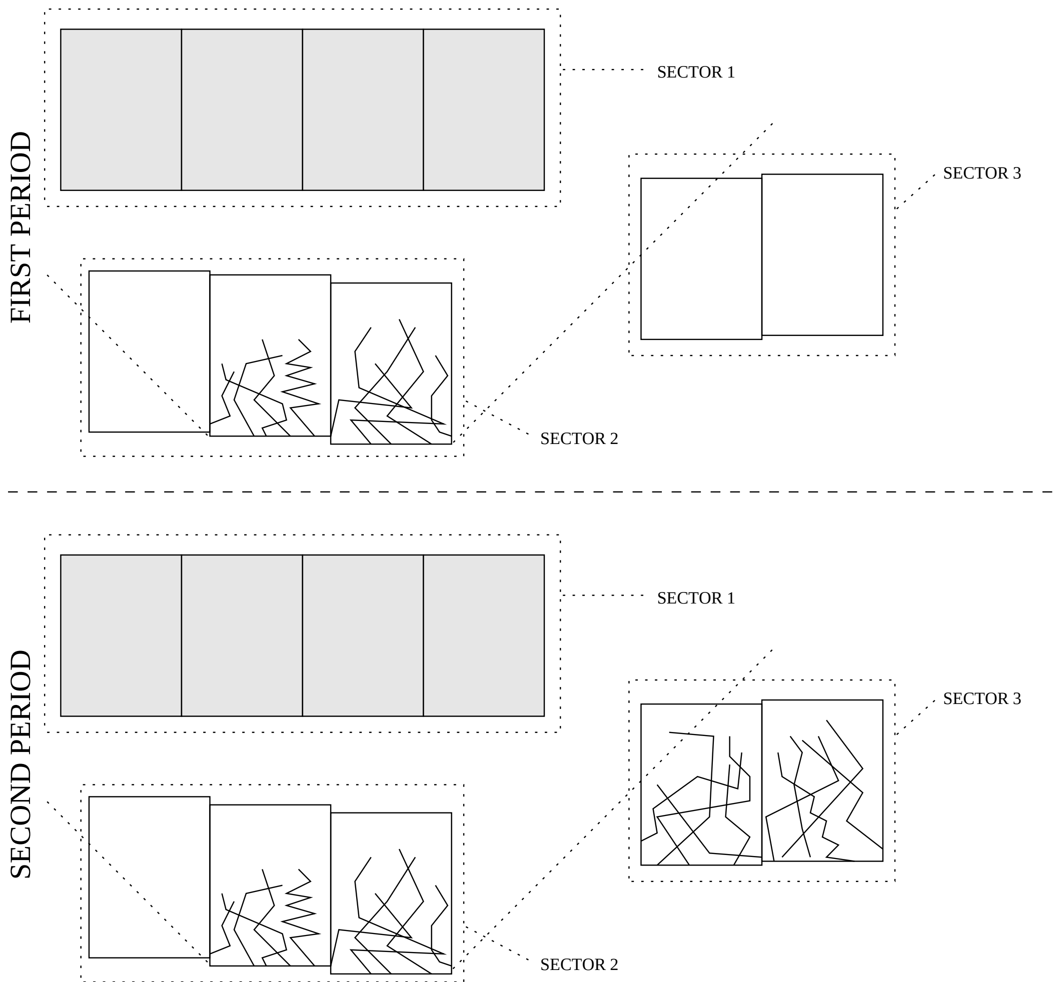

Figure 6: Levels exclusion.

In Figure 6, the entire SECTOR 1 (grey) becomes *forbidden* because of the exploitation of the two blocks in SECTOR 2, but blocks at SECTOR 3 are exploited in the next period. Notice that the presence of the constraint depends only on the base (floor) location.

Another constraint present in UM is that some connectivity is also desirable, in the sense that the exploitation of blocks which are not neighbors of another already exploited is expensive (either money or risk). To control it, the following rules have to be followed:

- Every block in the sector can be a *starting point*.
- The total number of starting points in a given sector cannot be superior to a fixed small number (1 or 2). Actually, the starting points may be fixed in advance.
- If a block is not a starting point, it can be exploited only if one of its neighbors has been exploited before.

Of course, capacity constraints force the inclusion of opportunity cost considerations in UM planning also.

2.2.1 A model

For each block i, the following information has to be available [5]:

- An economic value $v_i \in I\!R$, and a weight $w_i \in I\!R^+$,
- The set U_i of the blocks affected by its sinkage,
- N_i, the set of its neighbors.

There are also $\mathcal{S} = \{S_g\}_g$, the set of sectors, and for a given sector S_g, its maximum number of starting points $s_g \in I\!N$.

The global parameters of this model are: an horizon of T periods, a discount rate of $\alpha \in (0,1)$, and a capacity C_t for each period.

i, j denote blocks, g sectors and t, s periods, respectively.

So, for the variables

$$x_{it} = \begin{cases} 1 & \text{if block } i \text{ is exploited at } t, \\ 0 & \text{if not,} \end{cases}$$

and

$$f_i = \begin{cases} 1 & \text{if block } i \text{ is an starting point,} \\ 0 & \text{if not,} \end{cases}$$

the problem of sequencing an underground mine reads:

$$
\begin{aligned}
\max_{x_{it},\, f_i} \text{NPV} &= \sum_{i,t} \alpha^t \, v_i \, x_{it} \\
x_{it} &\leq f_i + \sum_{j \in N_i} \sum_{s<t} x_{js} \quad (\forall i)(\forall t) \\
\sum_{i \in S_g} f_i &\leq s_g \quad (\forall g) \\
\sum_i w_i \, x_{it} &\leq C_t \quad (\forall t) \\
\sum_{s \leq t} x_{is} &\leq \sum_{s \geq t} x_{js} \quad (\forall i)(\forall t)(\forall j \in U_i) \\
\sum_t x_{it} &\leq 1 \quad (\forall i) \\
x_{it}, f_i &\in \{0,1\} \quad (\forall i)(\forall t).
\end{aligned}
$$

Several extensions can be done to this model. An important one is to allow that portions of the blocks become exploited at each period.

An analogous prove to the one used for showing that the open pit mining sequencing is NP-Hard [8] can be applied to prove that this problem has the same complexity.

3 Stochastic elements

There are several sources of uncertainty in the mine planning business. The ones considered here are:

Block model errors: Measuring grades and the rest of the information to build the block model is expensive, so only a limited number of drillings (explorations) are performed to measure these variables. The rest of the data is estimated by statistical methods. However, it is known that these methods' estimations introduce some important errors.

Prices: The copper price can also vary, depending on the world copper needs.

Technology: Technology affects costs (decreasing them over time).

Accidents: Sometimes, a block remains *hanged*, the fractured rock could reach an equilibrium state and material does not flow. As well, it could happen that pit walls collapse, avoiding this sector to be exploited. Both situations mean that the mine scheduling has to be modified, affecting scheduling and company's revenues.

3.1 Robustness

The major goal of paying attention to such a probabilistic events is to build more robust plans for the mine exploitation. Currently, most of these elements are treated only doing some analysis based on planner's intuition, knowledge, priorities, experience, etc. Anyway, at the end of the day the result is a *good* plan for the planner's fixed assumptions.

But, what is robustness? Inspiration to answer this question can be found in [2]: *"Nature is unconcerned with achieving peak performance on any given problem. Natural genetics has evolved so the same search procedures could function under many different kinds of environmental conditions. This breadth combined with relative–if not peak–efficiency defines the primary theme of genetic search: robustness."*

As well as in nature, robustness in mining planning is related with survival. Changes in the price, variations in the demand, and (particularly) overestimated grades can make a project become unprofitable, which is like its death (or extinction, if the parallelism with evolution is kept). This means that robust plans have to be *flexible*, and preserve some *open doors* in the case that things do not go as expected.

4 Some elements of stochastic programming

The two reasons to introduce stochastic programming as a tool for mining scheduling are

- It seems to be better for large scale problems, because classical decomposition methods can be applied to improve model solving speed [1].
- As this and the next section will illustrate, the solutions of stochastic programs are implicitly robust.

Stochastic programming [1, 7] deals with the search of optimal decision under uncertainty presented in the next schema:

1. Some set of decisions x are made today.
2. During the night, some random events ω happen.
3. At the next morning, corrective actions y can be performed.

Variables represented by x are known as *first stage* variables, while y is the vector of *second stage variables* or just *resource*.

The most emblematic example of this kind of problems is the so-called *Farmer's problem* [1]. Here a farmer specializes in raising wheat, corn and sugar beets and has to decide how much land to devote to each crop, but depending on the weather, the productivity of the land changes. Also, at the end of the year, he can sell or buy any of the crops, but he has to be sure that will cover some minimum requirements of wheat and corn for cattle feed.

In this problem the first stage decisions x are the amounts of land devoted for each culture; the random elements are the yields for each product; and finally, the corrective decisions y (which are called *resource*) correspond to the selling/buying amounts needed to satisfy minimum production requirements and maximizing farmer's income.

In more mathematical terms, the so-called two-stage linear stochastic program with fixed resource reads:

$$\begin{array}{lll} \text{(TSLSP)} & \min & c^T x + \mathcal{Q}(x) \\ & s.t. & Ax = b \\ & & x \geq 0, \end{array} \tag{1}$$

where

$$\mathcal{Q}(x) = \mathbb{E}_\omega \left[\min\{ \, q^T(\omega) y(\omega) \mid W y(\omega) = h(w) - T(\omega) x, y(\omega) \geq 0 \, \} \right], \tag{2}$$

is called the *recourse function*, and $\omega \in \Omega$ are the possible scenarios (according to some probability space $(\Omega, \mathbb{P})$).

5 Mining scheduling models with stochastic programming

At this point the current models for (separated) underground and open pit mines have been presented. Besides, the concept of robustness and some elementary stochastic programming elements are already explicit also. So, it is time to put all things together. However, one additional thing will be done: the models are going to be developed for the case of a *combined mine*, that is, a mine using either underground or open pit technologies. The idea, of course, it is not to develop two separate models, run them by their own and after that compare the results to make the final technological decision. Instead of that, the model should allow the coexistence of both technologies, and to find an *optimum mixed schedule*.[2] To set up such a combined (or mixed) model obeys the facts that:

- Because of the increase in the amount of waste that must be removed in open pit mines, to pass from this method of exploitation to the underground one could be a good option to keep mines alive. Therefore, models allowing the coexistence of both technologies are a powerful tool to decide the right moment to do this technological change, or to find the way of balance them properly.
- It is easy to adapt the ideas exposed here to the classical cases of a one-technology mine.

In this section two approaches to set up mining scheduling models based on stochastic programming are presented. The first of them corresponds to a multistage stochastic binary

[2] Robustness is implicit here.

linear program with variables of the same kind of the previous (current) models. That is: deciding if some specific portion of the mine has to be exploited in some specific period of time, or not. The second model is quite more simple than the first one. It assumes that the plan is fully decided in advance (with first stage variables), then the random events take place, and finally the corrective actions in the schedule are done.

These models directly challenge the current ones because:

- The objective function is *the same*: The difference is that in the case of the stochastic models, it is not possible to maximize the net present value, but its expectation.
- As an intrinsic property of the modeling, solutions of the stochastic models are expected to be more robust. They are not fixed plans, but *policies* defining how to act in the different possible scenarios [7], while the current models give fixed solutions that are *recalculated* yearly on the base of the *new* future expectations.

Actually, the two stochastic linear models presented are generalizations of the current models, with the advantage of looking for more robust plans, because the solutions are more flexible: they have second stage variables to correct the plan depending on the events, as well as first stage decisions made looking at these future possibilities. On the other hand, it is expected that the results obtained with these models, measured only in the objective function terms, will be poorer, but that is because the values obtained in this way are, indeed, more realistic than the ones coming from the original models.

There are also some important disadvantages and problems with setting up these stochastic programming models. They are discussed during the formulation and also summarized at the end of the section.

5.1 Some general issues

The first problem to be solved is notation. In the two-technologies model, either *block* or *cell* are used for naming blocks of the block model, and the big blocks of underground mining will be called *macroblocks*. These macroblocks are grouped into *sectors*.

The second point before setting up a model is to say how technologies, uncertainty sources and stochastic programming will take part in the modeling.

5.1.1 Treatment of the uncertainty sources

Prices are not an explicit part of the model, but they affect the economic values of blocks and macroblocks, then different possible scenarios are considered in the estimation of those values. The same applies to the uncertainty coming from the block model: despite in the real world it is possible to *buy information* doing more explorations to improve estimations, this option is not considered in the modeling.

About the technological variations, they will not be included, because there exists the policy that the results (NPV values) must be reachable with proven (existing) technology, in order to be compared with each others.

Finally, even though accidents are possible, experts hold that their effect is not significant in the long term. Thus, they are not going to be included in the model.

5.1.2 Re-blocking

To reduce the amount of variables corresponding to open pit, a technique called *Re-blocking* will be used. Basically, re-blocking corresponds to group blocks into bigger ones, allowing a more aggregated problem to be considered.

5.1.3 Some assumptions

Technological exclusion: For simplicity, a macroblock level of exclusion will be supposed for the employment of both technologies. It means that they cannot be applied at the same macroblock. This is more restricted than the real world, where this exclusion is posed at the block level.

Preprocessing: Re-blocking, macroblocks and sector are given, not only in their position and dimension, but also with their economic valorization.

5.2 A first model

Modeling is based on a block/macroblock model as shown in Fig. 7 (but not necessarily fitting exactly as appearing there).

It means that two "block models" have to exist, a set of blocks (indexed by i and j), and a set of macroblocks (with indices m and n). There is also an horizon T, so $1 \leq s, t \leq T$ are indices reserved for time periods.

5.2.1 Variables

For each block i and period t, let be the next variables related to open pit:

$$r_{it} = \begin{cases} 1 & \text{if cell } i \text{ is able to be exploited since } t \text{ by open pit technology,} \\ 0 & \text{if not,} \end{cases}$$

and

$$p_{it} = \text{Percentage extracted from } i\text{-th block at period } t.$$

On the other hand, for underground mining

$$u_{mt} = \begin{cases} 1 & \text{if macroblock } m \text{ is exploited at time } t \\ & \text{by underground methods,} \\ 0 & \text{in other case,} \end{cases}$$

and

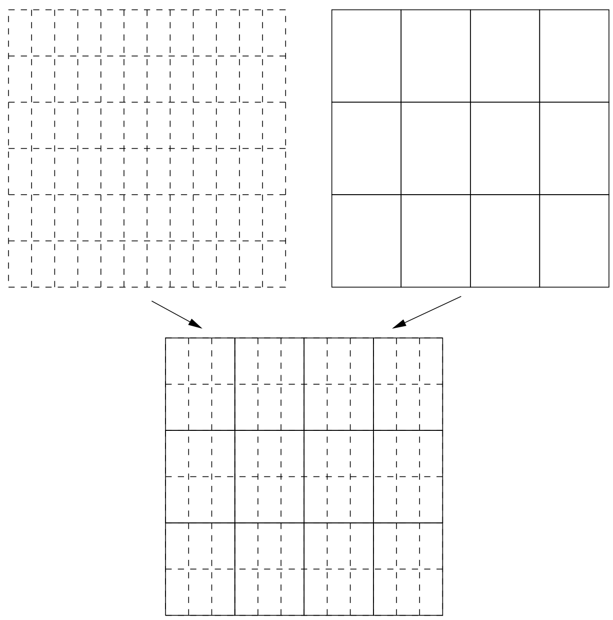

Figure 7: Block/Macroblock model for mixed mining.

$$f_m = \begin{cases} 1 & \text{if block } m \text{ is an independent starting point,} \\ 0 & \text{otherwise,} \end{cases}$$

are defined. It is also useful to consider:

$$P_{it} = \sum_{s \leq t} p_{is}, \quad U_{mt} = \sum_{s \leq t} u_{ms},$$

which respectively represents the accumulated percentage exploited from block i at time t, and if macroblock m has been exploited in the interval $1, \ldots, t$.

Notice that the only first-stage variables are the starting focuses ones f_m. The rest of them are stochastic and depend on the previous stages. For instance

$$p_{it} \;=\; p_{it}\big(p_{js}, v_{is}, w_{is}, u_{ms}, v_{us}, w_{ms}\big)_{s=1}^{t}.$$

5.2.2 Stochastic variables

In order to set the stochastic programming model (which is going to be a multistage one), it is not possible to use random variables such as v_i and v_m for economic values and weights w_i and w_m, respectively. The problem with this is that only the exploited blocks will reveal their real economic value and weight. This means that, if the random coefficients are chosen as just explained, the variables in the problem are selecting which random coefficients are realized, which is not allowed by the stochastic programming framework. One way to solve it is to select the random coefficients as follows:

- v_{it} being the economic value of block i at time t,
- v_{mt}: the economic value of macroblock m at time t,
- w_{it}, which is the weight of block i at time t,
- w_{mt}, the analogous variable for macroblock m.

However, it forces distributions strongly depending on the decision variables. More precisely, if $\mathcal{V}_i$ is the distribution for the economic value of block i, then for every $t > 1$

$$v_{it} \sim P_{i\,t-1} v_{it-1} \;+\; (1 - P_{i\,t-1})\mathcal{V}_i, \tag{3}$$

that is: $v_{it} = v_{is}$ if the block has been exploited at any time $s < t$, or it will take a random value following a distribution $\mathcal{V}_i$ if not. Of course, similar definitions must be carried for the rest of the random coefficients.

5.2.3 Deterministic data and parameters

Several definitions are required in order to write the constraints and set up the model. At first, let

$$I_m = \{i \mid \text{block } i \text{ intersects macroblock } m\}$$

As well, the sets N_i and N_m of the neighbors of block i and macroblock m respectively are defined.

About level exclusion, it concerns both technologies: The sinkage of macroblock m affects the set U_m^U of macroblocks over it and the set U_m^P of blocks located inside its cone of influence.

As usual, a discount rate of α and an horizon T are considered, thus s, t represent periods.

Also, given two neighboring *columns* of blocks, the difference between their heights have to be bounded by a percentage δ of an unitary block's height (blocks are considered homogeneous in their dimensions).

About the capacities, there are three involved at time t:

- The capacity of open pit exploitation C_t^P,
- C_t^U: limiting underground mining and,
- The global plant capacity C_t.

Finally, if i is a block then i^- denotes the block immediately over i, $K \in \mathbb{N}$ is the maximum number of active macroblocks at each time, as well as $\mathcal{S}_g$ and s_g define, respectively, sectors and their maximum number of starting points.

5.2.4 Objective function

Only for the sake of clear and short writing, let

$$F_t = \sum_i v_{it} p_{it} \; + \; \sum_m v_{mt} u_{mt},$$

the cash flow at period t. The function to be maximized is the expected NPV

$$V \; = \mathbb{E}\Big[F_1 + \alpha \mathbb{E}\Big[F_2 + \alpha \mathbb{E}\big[F_3 + \alpha \mathbb{E}[\cdots] \,|\, \mathcal{I}_2\big] \,|\, \mathcal{I}_1\Big]\Big]. \tag{4}$$

Here $\mathcal{I}_t$ represents the *information collected up to time* t, that is, the variables p_{is} and u_{ms}, as well as the random coefficients v_{is}, w_{is} and v_{ms}, w_{ms} for each $s \leq t$.

5.2.5 Constraints

These can be classified into several groups:

Elementary constraints

Blocks and macroblocks can be exploited only once:

$$(\forall i) \qquad \sum_t p_{it} \ \leq \ 1 \tag{5}$$

$$(\forall m) \qquad \sum_t u_{mt} \ \leq \ 1. \tag{6}$$

Open pit exploitation constraints

Here precedence constraints are set as follows. First of all, a block must be exploitable:

$$p_{it} \ \leq \ r_{it} \ (\forall i)(\forall t), \tag{7}$$

but, to be exploitable, the cell right over itself must be extracted completely before:

$$r_{it} \ \leq \ \sum_{s \leq t} p_{i-s} \ (\forall i)(\forall t). \tag{8}$$

Adjacent cell exploitations have to be similar

$$(\forall i)(\forall t)(\forall j \in N_i) \ \sum_{s \leq t} p_{is} - p_{js} \ \leq \ \delta, \tag{9}$$

there are capacity bounds to be satisfied

$$(\forall t) \ \sum_i w_{it} \, p_{it} \ \leq \ C_t^P, \tag{10}$$

and finally, the next constraint imposes level exclusion for the open pit method

$$\sum_{s \geq t} p_{is} + \sum_{s \leq t} u_{ms} \ \leq \ 1 \ \ (\forall m)(\forall s \in U_m^P)(\forall t). \tag{11}$$

Underground exploitation constraints

Macroblocks can be exploited if they are starting points or one of its neighbors has been exploited before:

$$u_{mt} \ \leq \ f_m + \sum_{n \in N_m} \sum_{s < t} u_{ns} \ \ (\forall m)(\forall t). \tag{12}$$

The number of starting points is bounded in each sector as follows:

$$\sum_{m \in S_g} f_m \ \leq \ s_g \ \ (\forall g). \tag{13}$$

Level exclusion exploitation may be imposed as:

$$\sum_{s \geq t} u_{ms} + \sum_{s \leq t} u_{ns} \ \leq \ 1 \ \ (\forall m)(\forall n \in U_m^U)(\forall t). \tag{14}$$

Additionally, the number of macroblocks being exploited simultaneously is bounded:

$$(\forall t)\ \sum_m u_{mt} \ \le\ K. \tag{15}$$

Finally the capacity constraint reads:

$$(\forall t)\ \sum_m w_{mt}\, u_{mt} \ \le\ C_t^U. \tag{16}$$

Common/shared constraints The plant limitation is written as

$$(\forall t)\ \sum_i w_{it} p_{it}\ +\ \sum_m w_{mt}\, u_{mt} \ \le\ C_t, \tag{17}$$

and the technological exclusion reads

$$(\forall m)(\forall i \in I_m)\ U_{mT} + P_{iT} \ \le\ 1. \tag{18}$$

5.2.6 Observations of the first model

As an extension of the currently used programs, model (4) - (18) represents properly the problem it was set up for: to establish feasible exploitation sequences to obtain high revenues for the company.

From the stochastic programming theoretical point of view, things are not that good. At first, the model is not a *fixed resource problem*, because the matrix multiplying the second stage decision variables is stochastic. Besides, defining random coefficients for each structure and period does not seem natural for a stochastic program. On the other hand, such distributions are very difficult to build properly, because of (3) and the relationships that have to exist: in a geometrical way (close structures have related economical values), and in a temporal one (if the copper price changes, it changes for the not sold blocks and macroblocks).

5.3 A second (simpler) model

A better approach (from the theoretical side at least) can be done if the next schema is considered:

1. The full sequence of exploitation is fixed,
2. All the random variables are realized,
3. Some corrective actions are performed.

An interesting observation is that the only possible in-feasibility arising when the random variables are finally known is to exceed the capacity. However, this mathematical problem is *good* for the business, because it means that the mine is richer than it was expected (if the block weights are bigger than ones waited for, it is because there is more ore to be processed).

The simplicity of the model gives some hope of proving properties as *completeness*[3] of the resource, and then, on the treatability of the problem. This treatability is reaffirmed by the fact that the resulting model is a two-stages stochastic program (and not a T-stages one).

On the other hand, there seem to be three possibilities about the second stage's decisions:

- To have a complete set of new variables, let us say p_{it}^2, r_{it}^2 and u_{mt}^2, to confirm, reject or add new possible extractions. In this case there should be a limit on the number of changes (to avoid a complete full-information rebuilding of the plan).
- Allowing only regretting first-stage exploitation decisions, in order to fix the possibility of unsatisfied capacity constraints. This has the advantage of permitting partial decreasing of the exploitation of macroblocks, which means a continuous second-stage program, and thus, a simpler program.
- A *combination* of the first two options: to allow some kind of options for the open pit part, and others for the underground mine. It arises, for instance in the physical nature of the problem: it seems difficult to increase the size of the pit without intersecting the underground mine.

An important problem with this model is, however, that it is not very accurate to the way mining is done, because there is not a way to know all the random elements in one shot. Then, the solution found should be understood not like a final plan but as giving some guidance and information about robust/conservative policies to be applied to the exploitation of the mine.

5.4 Summary

Current models for mining scheduling are generalized using stochastic programming to set up new models with the advantage of considering the possible random events to elaborate robust schedules, this is, more flexible solutions minding different possible scenarios for the future. The preexisting models are also generalized in the sense that the new models can be applied to a mine combining underground and open pit technologies optimally.

On the other hand there are some difficulties using stochastic programming for mining scheduling. In the first model, it does not seem to be easy at all to set accurate distributions for the random elements in the problem. The second model is quite simple and successfully avoids the problem just mentioned, but the solution obtained needs to be studied deeper,

[3] A stochastic program has complete resource if for every pair x of first stage decisions and every realized ω, there is a vector $y(\omega)$ of corrective actions that makes the solution $(x, y(\omega))$ feasible.

because the random elements in the model are realized in one shot, which is not the way things happen in the real world. Of course, any of the two models presented will need more computational efforts than the current ones to be solved.

To finish this section in a more optimistic fashion, it is important to mention several *sub-products* that can be extracted from the ideas presented here:

- If all the stochastic elements are omitted, the result is a model allowing either underground and open pit technologies to coexist optimally (from the NPV point of view, at least).
- If the open pit part is *deleted*, the result would be stochastic programs for underground mining. As well, an open pit stochastic program is obtained when the underground part is removed.
- Continuous variables describing block's extraction after re-blocking is new. Before this work, re-blocking has been just to group blocks into bigger ones and then *forget* the original blocks, introducing an important loss of precision making the treatment of the collapsing angles difficult, which is avoided with the continuous approach.
- Robustness in mining scheduling is also a new concept, which appears as an implicit feature of the stochastic programming solutions.

6 Demand/Capacity models

In this section, a new kind of model is exposed, motivated in a deeper revision of the current models in order to rescue the most relevant elements from the robustness point of view.

A first important question arising is: *Why the revenue for the company due to the ore extracted at some instant is the value of selling it at this time?* Actually, there exists the possibility to save (and not sell) extracted mineral in order to influence the price. This means that some selling policies exist, and then, the moment when the ore is extracted does not coincide with the instant of its sale.

The second element also appears in the objective function: The discount rate. This element introduces *greed* in the exploitation plans, reducing mine's life, and that does not sound like robustness at all. Actually, at this point, one realizes that the central planner priority is not necessarily the economical value. Surely it is an important issue, but the social and political costs of closing a mine are huge, so, again, greedy schedules reducing the life of the mine seem undesired.

The new approach considers the first point as imposing some minimum requirements (demands) of ore at each period, because that avoids the *"sell it as soon as you got it"* false principle, allowing the planner to introduce stock and selling policies implicitly when fixing such minimum exigencies.

The second point is carried by the objective function, which in this approach is not the NPV of the sequence, but the amount of the residual ore in the mine when the horizon T

is achieved: the one which has not been exploited. Thus, the final problem is to satisfy the demands with "minimum cost" in the sense that as much as possible ore is saved for the future. In fact, the ore is saved *in situ*, thus there is no stocking cost to be paid.

High revenues are subordinated to the satisfaction of the demands at each period instead of maximizing them directly (or vice versa: to have the possibility of selling more, the planner only needs to increase the demands). In this sense, problem's feasibility will depend on the planner's greed: the higher demands imposed, the more difficult to satisfy them, specially if the mine does not result to be as rich as expected.

6.1 Mathematical formulation

Because of the not explicit presence of NPV in the model, not economical values for the structures are needed. Instead of that, let us consider coefficients v_i representing the *pure ore* contained by i.

As in the case of NPV-models, it is possible to write a two-stages model or a T-stages one. Nevertheless, demand constraints make the problem of second stages uncertainty quite most difficult to be solved, eliminating the good theoretical properties expected for a stochastic approach. For that reason, only a deterministic model is presented.

6.1.1 Variables

The variables are the usual: $r_{it} \in \{0,1\}$ representing if the block i is ready to be exploited or not at time t, $p_{it} \in [0,1]$ being the percentage exploited from this block at the t-th period, and $u_{mt} \in \{0,1\}$ is the decision of exploiting (or not) the macroblock m at the instant t. As well, the starting points variables, f_m stay unchanged.

6.1.2 Data and parameters

For the cell i, let bet v_i and w_i the coefficients representing the pure ore and waste amount present at this block, and v_m the content of ore in macroblock m. All these quantities are measured in tons.

As in model (4)-(18), let be I_m: the set of blocks intersecting the m-th macroblock.

The neighborhoods' definitions are also preserved: N_i and N_m as the sets of neighbors of block i and macroblock m respectively.

Level exclusion is established via the set U_m^U of macroblocks over it and the set U_m^P of blocks located inside its cone of influence, as well as there is a tolerance of δ for adjacent pit columns.

As above mentioned, in this model there are demands and capacities for time t:

- D_t: The demand in ore tons at time t,
- C_t^P: The extraction capacity for the open pit mine at time t,
- C_t^U: Corresponding coefficient for underground mine,

- The global plant capacity C_t.

Finally, as before, i^- denotes the cell immediately over i and K bounds the number of active macroblocks.

6.1.3 Objective function

The objective function is the remaining ore (the not exploited one), which can read as:

$$\begin{aligned} S &= \sum_i v_i + \sum_m v_m - \\ &\quad \left[\sum_{i,t} v_i \, p_{it} + \sum_{m,t} v_m \, u_{mt} \right] \end{aligned} \tag{19}$$

where the first term is the total ore contained in the mine (a constant number), and the second is the addition of the extracted via open pit and underground mining, respectively.

This is not the only choice. Others options for the objective function are:

- To give some preference to the *ready to be exploited ore*, that is, the one for which there is warranted access at T but it has not been exploited yet. *A fortiori* this quantity can be written linearly in the current variables.
- The *not exploited and available ore*, to force the model to mind that an not very fortunately exploitation of a macroblock can leave big portions of the mine unable to be exploited. To do that, the trick is to consider one additional period of time, without capacity constraints, and maximize the ore exploited then.

6.1.4 Constraints

As done before, they are separated in:

Elementary constraints

Blocks and macroblocks can be exploited only once:

$$(\forall i) \qquad \sum_t p_{it} \;\leq\; 1 \tag{20}$$

$$(\forall m) \qquad \sum_t u_{mt} \;\leq\; 1. \tag{21}$$

Open pit exploitation constraints

Precedence constraints are set as follows. First of all, a block must be exploitable:

$$p_{it} \;\leq\; r_{it} \; (\forall i)(\forall t), \tag{22}$$

but, to be exploitable, the cell right over itself must be extracted completely before:

$$r_{it} \;\leq\; \sum_{s \leq t} p_{i-s} \;(\forall i)(\forall t). \tag{23}$$

Adjacent cells exploitation have to be similar

$$(\forall i)(\forall t)(\forall j \in N_i)\; \sum_{s \leq t} p_{is} - p_{js} \;\leq\; \delta, \tag{24}$$

the capacity bound have to be satisfied

$$(\forall t)\; \sum_{i} (v_i + w_i) p_{it} \;\leq\; C_t^B, \tag{25}$$

and finally, the next constraint imposes level exclusion for open pit method

$$\sum_{s \geq t} p_{is} + \sum_{s \leq t} u_{ms} \;\leq\; 1 \;\; (\forall m)(\forall s \in U_m^P)(\forall t). \tag{26}$$

Underground exploitation constraints

Macroblocks can be exploited if they are starting points or at least one of its neighbors has been exploited before:

$$u_{mt} \;\leq\; f_m + \sum_{n \in N_m} \sum_{s < t} u_{ns} \;\; (\forall m)(\forall t). \tag{27}$$

The number of starting points is bounded in each sector as follows:

$$\sum_{m \in S_g} f_m \;\leq\; s_g \;\; (\forall g). \tag{28}$$

Level exclusion exploitation may be imposed as:

$$\sum_{s \geq t} u_{ms} + \sum_{s \leq t} u_{ns} \;\leq\; 1 \;\; (\forall m)(\forall n \in U_m^U)(\forall t). \tag{29}$$

The number of macroblocks being exploited simultaneously is bounded:

$$(\forall t)\; \sum_{m} u_{mt} \;\leq\; K. \tag{30}$$

Finally the capacity constraint reads:

$$(\forall t)\; \sum_{m} w_{mt} u_{mt} \;\leq\; C_t^U. \tag{31}$$

Common/shared constraints

The technological exclusion is:

$$(\forall m)(\forall i \in I_m) \sum_t u_{mt} + p_{it} \leq 1, \tag{32}$$

similarly to the original formulation, the plant capacity restriction reads:

$$(\forall t) \sum_i v_i \, p_{it} + \sum_m v_m \, u_{mt} \leq C_t, \tag{33}$$

and, of course, the demand constraint:

$$(\forall t) \sum_i v_i \, p_{it} + \sum_m v_m \, u_{mt} \geq D_t, \tag{34}$$

6.2 Models comparison

- The capacity approach is more realistic in the sense that it does not assume that ore is sold at the same time it is exploited. This allows the central planner to establish some selling policies and to adapt the exploitation to them, or vice versa.
- There is a compromise with the life of the mine, because future (what happens after the horizon T) is considered in the decision process. For this reason, this kind of modeling is more robust than the NPV oriented ones.
- The aim of the model is to provide some minimum warranties (to satisfy demands), which (again) is a point in favor of the robustness of these models.
- Plant's size and its relationship with the mine itself appears clearer in demands terms.
- It is possible (for instance) to *study* the mine, in the sense of *how much it can give*: the higher the demands, the more difficult to satisfy them, so infeasibility can accuse the maximum exploitation rate of the mine.
- While in the NPV models the costs and benefits are explicit, in the demand/capacity models all the economic elements are implicit in the demands and capacities of exploitation.

7 Solving the demand/capacity model

In this section a simple heuristic is developed in order for solving the demand/capacity model. This heuristic tries to solve the instances quickly by reducing the size of the problem. It has two major steps.

7.1 Filtering

The motivation of this part comes from the next:

1. Solve the original problem and call z^* its solution (this requires re-arrange the variables). For simplicity, denote z_t^* the *sub-vector* containing the variables of the t-th period.

2. Add the solution variables over t, and call $\bar{z}$ the result of this operation
$$\bar{z} = \sum_t z_t^*.$$

3. Now consider the same problem but with only one period, and demands and capacities set to the total demand and capacities in the original problem. Solve it and name its solution $\bar{z}^*$.

Because $\bar{z}$ is feasible to the second problem, it follows that $\bar{z}^*$ reaches a better objective value, that is, leaves more not extracted ore in the mine. Indeed, if a partition of the exploited ore by $\bar{z}^*$ into periods satisfying the demands and capacities is found, then this partition will be, not only feasible to the first problem, but optimal.

After this argument is presented, the first part of the heuristic is not quite surprising:

- Reduce the problem to a one-period instance by aggregating demands and capacities, solve it, and call z^* its solution.

- Delete the not exploited part of the mine, that it, all blocks and macroblocks with associated values of z^* being zero.

7.2 Periods partition

Once the first part is done, its solution z^* determines the portion of the mine to be considered in this step of the algorithm. This means that:

- The size of the problem is reduced by deleting *not interesting* parts of the mine.

- A sequence of exploitation still must be found.

- Now, there is not sense in minimizing the extracted ore.

The last point gives a way to reduce (again) the size of incoming problem of extracting a valid sequence from z^*: to put part of the constraints in the objective function, penalizing infeasibility. At least two options can be considered to do it: the demands' constraints and the capacity ones. These choices are, in some sense, complementary: while preserving the capacity restrictions while *spread* the extraction over the periods (granting some *minimum* exploitation amounts at each one), the analogous occurs by relaxing these constraints, because to satisfy demands should separate exploitation in periods, avoiding huge exploitations and, then, helping the capacity constraints to be hold. Of course, in both cases there is not warranty of feasibility, and that is why this is only a heuristic.

Notice that the complementariness just mentioned is not necessarily valid in the original problem where both constraints are fundamental. For instance, if one avoids the capacity constraints in the original model, then it is possible that the solution asks to extract the most poor zones of the mine, satisfying demand's constraints by extracting a lot of zones simultaneously, which shall be infeasible because of the costs involved.

Of course, it is possible that (even after the relaxation) the problem becomes infeasible:

- Unsatisfied demands could arise due to macroblocks indivisibility: if the ore contained in one of them is required to satisfy two different periods, the model will not allow it. A tricky solution which could be used to solve this is to have some slack demand at the first stage, so it can be available in the second. On the other hand, it is also necessary to remember that indivisibility of macroblocks is just a model constraint, that does not hold in real world.
- Capacity constraints can also become unsatisfied, due to an analogous reason: there could be a macroblock rich enough to not have a period where it can be exploited without breaking capacity constraints. However, this is not quite serious: it is not expected to occur frequently and when it happen, additional machinery and manual labor can be contracted. Indeed, this is a good reason to relax the capacity constraints at the objective function, because the cost of violating them is better known.

Finally, it is important to notice that:

- Since the simplified version in the first step and the original model have the same objective function, and every valid sequence defines an feasible solution of the one-period model, the objective value obtained by the heuristic can be used as a bound generator for the general problem.
- The heuristic also suggests an algorithm for solving the problem: to satisfy demands period to period, by separating the first one, and aggregating the rest in a second stage.

So the above-mentioned infeasibility problems are, indeed, not quite serious.

8 Conclusions

Current model's lack robustness is a problem that needs to be solved. This work presents two major approaches to achieve that:

Stochastic Programming: Solution to stochastic programs are implicitly robust because of their flexibility: first stage decision are made looking to *compatibility* with the future and the available possibilities to afford the uncertainty, so two models are developed inside this framework: a T-stages mixed linear model, and a 2-stages one. Both of them are direct generalizations of the current models but minding the uncertainty of the process, but while the first could be difficult to set up, the second needs a deeper study.

Demand/Capacity satisfaction: After a deeper revision of the current models, explicit economical considerations are removed (in the hope that new models become developed for studying them properly), but kept implicitly included via demands and capacity constraints. The resulting model is a deterministic one, enforcing robustness by: looking for plans satisfying demands and preserving as much as possible ore for future.

Because the second approach seems more promising, a heuristic is proposed to solve its instances, but it is still necessary to provide numerical results, specially over real data.

This work also exposes several interesting ideas that could be introduced in mining scheduling: robustness, combined mining and continuous reblocking are some examples of them.

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9 Appendix: Opportunity cost

Let consider the next example. For a one-month job, two payment options are given:

- Four payments, once per week, of \$200,
- Just one final payment of \$900.

Assuming that both options are feasible the question is: Which is the best choice?. Unfortunately, it is not as easy as comparing 900 with $200 \times 4 = 800$, because there is a "bank" that offers 10% of interest per week[4]. It means that saving the first salary, there will be $200 \times (1 + 10\%) = 220$ at the end of the second week, and at this moment it is possible to make a second depot for $220 + 200$, so at the end of the third week the bank will return: $440 \times 1.1 = 484$. Of course, again, one shall save in the bank this amount plus the third salary, getting a total sum of \$752, thus at the end of the month, the final amount will consist of \$952: that it is \$52 over the second option.

The preceding procedure compares the amount at the end of the month, but the same can be done measured in *today's money*. In this case it is necessary to convert both options to their *present value*:

- What is the today's equivalent amount for the \$900 at the end of the month? It corresponds to the amount of money x such that, if it is put in the bank for the 4 periods, it is going to be transformed into \$900, that it:

$$\underbrace{(1.1) \times \underbrace{(1.1) \times \underbrace{(1.1) \times \underbrace{(1.1) \times x_2}_{\text{after first week}}}_{\text{after week 2}}}_{\text{third week ending}}}_{\text{end of the month}} = 900$$

  then

$$x_{\text{one payment}} = \frac{900}{(1.1)^4} = 614.71$$

- Following the same rule, it is possible to convert each \$200 salary to is corresponding today's amount to get:

$$x_{\text{four payments}} = \frac{200}{(1.1)} + \frac{200}{(1.1)^2} + \frac{200}{(1.1)^3} + \frac{200}{(1.1)^4} = 635.35$$

The result, of course, is the same: the four payments' option is the best. However, that is the customary way to compare this kind of numbers (or projects, or business), and in the value obtained is called the *net present value* (NPV) which can be also understood as the minimum price one should accept for "selling" this "idea" or "project".

[4]And that is where the name *opportunity cost* comes from: choosing the one payment option means to pay a cost: the value of the chance to make some business with the money during the time one has to wait to receive our payment.

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